IMPROVING PERFORMANCE OF PID CONTROLLERS USING ADAPTIVE CONTROL

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Abstract

A simple Add-On adaptive control algorithm is presented that can improve the performance of any linear time-invariant plant controlled by PID controller. The algorithm is simple. It is proved that such algorithm always exists. The proof is constructive, that is, it is simple and straight forward to design and implement the control algorithm. The algorithm is presented as a set of equations and in a block diagram that demonstrate the simplicity of the algorithm. It is demonstrated that this approach completely solves the portability problem. The paper is oriented towards the practitioner and therefore any proof that already appears in the literature is only cited. An example demonstrates the improvement in performance.

KEY WORDS
Adaptive control, PID controller, ASPR.

1. Introduction

Control engineer often tackle the following problem: There is an existing system (e.g. target tracking problem, LOS stabilization, servo system, etc.) that is performing well and meets some specifications, i.e. stable, robust and with satisfying response. The engineer is required to devise for the same type of application a system with improved performance, e.g. smaller tracking errors, faster response, improved robustness, etc., under changing and uncertain environment.
There are several approaches to this challenge:
i) Redesign the whole system;
ii) Replace or add sensors;
iii) Adopt new motion and control technology;
iv) Apply algorithms with existing hardware; and more.
In this paper we adopt solution No. (iv), i.e. the following solution is proposed: An Add-On adaptive control algorithm that will supplement the existing control algorithms and thus improve the performance and hopefully meet the updated requirements.
This is a Cost-Effective approach as it does not require design and manufacture of new hardware.
This approach guarantees improvement in performance, improvement in robustness of performance under changing operating regime.
In previous paper [1] it has been shown that such algorithm always exists for stable systems. In this paper this result is generalized and it is shown and proved that such algorithm always exists for any system controlled by PID controller.
We are not replacing the existing PID control algorithm; therefore we assume that the existing system is stable with sufficient gain and phase margins.
The novelty of this paper is the proof that shows that there always exists an algorithm that improves the performance and robustness of the system. The presented algorithm completely solves the portability problem.
The Add-On adaptive control algorithm is based on the Simple Adaptive Control algorithm (SAC) [2,3].

2. Statement of the Problem

We assume that the existing system, Figure 1, is a two degrees of freedom (performance and robustness) and two Building block controller (trajectory generator and the closed loop PID controller. For the definitions of the nomenclature see [4]. The trajectory generator produces the trajectory that the closed loop system is required to track with the smallest tracking error possible, ideally zero. This system is stable, robust (gain and phase margins) and satisfies some performance requirements (time response, overshoot, steady state and maximal tracking error, etc.).

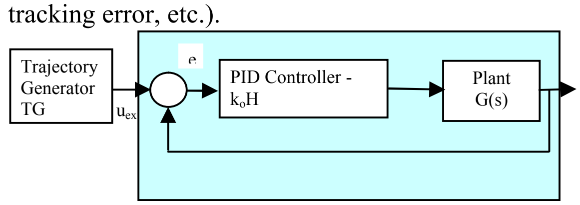

Figure 1: Existing System control architecture: a Two Building Block Controller architecture: One block- PID controller- koH(s), Second Block – Trajectory generator.

and we have

$$\boldsymbol{T}(\boldsymbol{s}) = \frac{\boldsymbol{k_o HG}(\boldsymbol{s})}{1+\boldsymbol{k_o HG}(\boldsymbol{s})} \qquad (1)$$

The problem is to design a system with improved performance.

3. The proposed solution of the problem – Add-On adaptive controller

In order to proceed and present the proposed Add-On Adaptive Controller we consider the following three

block realizations of the system depicted in Figure 1.
We prefer to operate on the existing closed loop for several reasons:

1. The original system is not changed (this is good for maintenance, the existing personnel is familiar with the equipment,…);
2. The uncertainty of the closed loop is reduced by the existing controller, therefore the added controller have to deal with less uncertainty;
3. The added controller can always be disconnected and the original performance restored; and more.

We partition the controller $k_oH(s)$ as presented in figure 2. It is assumed that $H_1(s)$ is the PID controller.

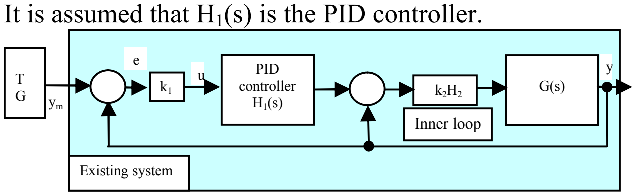

Figure 2: Three block realization of the Existing System control architecture.

We have

$$\tilde{T}(s) = \left. \frac{k_1 k_2 H_1 H_2 G(s)}{1 + (1 + k_1 H_1) k_2 H_2 G(s)} \right|_{k_1 >> 1} \cong T(s) \tag{2}$$

If k_1>>1 the two realizations, on Figure 1, eq. (1) and figure 2, eq. (2), respectively, are equivalent.
We assume that k_2 is such that inner loop

$$G_p(s) = \frac{k_2 H_2 G(s)}{1 + k_2 H_2 G(s)} \tag{3}$$

is stable and gives acceptable performance.
Then the proposed Add-On Adaptive algorithm control architecture is as presented in Figure 3.

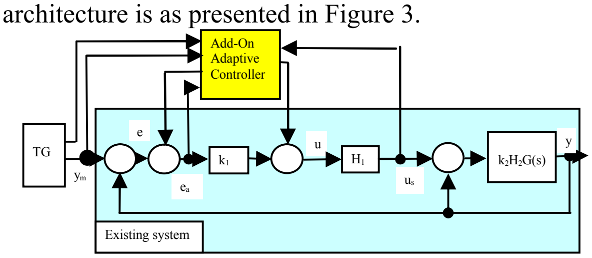

Figure 3: Proposed Add-On Adaptive algorithm control architecture.

4. The Add-On Control Algorithm for SISO plants

We treat here only SISO systems the sake of simplicity. The algorithm for SIMO, MISO, MIMO systems are described in the literature and are simple augmentation of the algorithm for SISO algorithm [2,3]. The necessary conditions for the performance of the algorithm are presented in following sections.
Let us assume that the system G(s) in closed loop with k_2H_2 is strictly proper system of any order and

$$P(s) = \frac{Y(s)}{U(s)} = H_1 G_p = H_1 \frac{k_2 H_2 G}{1 + k_2 H_2 G} = C(sI - A)^{-1} B \tag{4}$$

The PID controller $H_1(s)$ is

$$\begin{aligned} H_1(s) &= \frac{U_s(s)}{U(s)} = k_p + \frac{k_I}{s} + \frac{s k_d}{s\tau_d + 1} \\ &= \frac{s^2(k_d + k_p\tau_d) + s(k_p + k_I\tau_d) + k_I}{s(s\tau_d + 1)} \end{aligned} \tag{5}$$

The plant $G_p(s)$ (3) whose order may be very large and unknown, with the PID controller (5) in cascade, That is $P(s)$ (4), is required to follow the input-output behavior of the (arbitrary) model reference that in the motion control literature is called also Trajectory Generator(TG) represented by

$$\begin{aligned} \dot{x}_m(t) &= A_m x_m(t) + B_m u_m(t) \\ y_m(t) &= C_m x_m(t) \\ M(s) &= C_m (sI - A_m)^{-1} B_m \end{aligned} \tag{6}$$

where $x_m \in R^m$, $y_m \in R$, $u_m \in R$.
It is worth mentioning that in this approach the so-called model is only a trajectory generator. This trajectory generator is only used in order to define, or "model," the desired input-output behavior of the plant, but it is free otherwise and is not a result of some reduced-order modeling of the plant (and, in particular, m<<n is permitted).
The tracking error is

$$e(t) = y_m(t) - y(t) \tag{7}$$

For the realization of the Add-On algorithm a building Block, the Parallel Feedforward Configuration (PFC) denoted $D(s)$ is required. The rational behind this building block is presented shortly in the following sections and in depth in the references [3].
Realization of the PFC is

$$\begin{aligned} \dot{s}(t) &= A_s s(t) + B_s u_s(t) \\ y_s(t) &= C_s s(t) \end{aligned} \tag{8a}$$

$$D(s) = \frac{Y_s(s)}{U_s(s)} = C_s (sI - A_s)^{-1} B_s \tag{8b}$$

The selection of $D(s)$ is as follows: Select some transfer function $Q^{-1}(s)$:

i) that is minimum phase(all zeros are in the LHP);
ii) has relative degree one(one more poles than zeros)
iii) has no poles on the imaginary axis;
iv) if $Q^{-1}(s)$ is also stable then $D(s)$ is stable as well;

then

$$D(s) = Q^{-1}(s) - G_p(s) \tag{9}$$

The augmented error, with slight abuse of notation, is

$$\begin{aligned} e_a(t) &= y_m(t) - y(t) - D(s) u_s(t) \\ &= e(t) - D(s) u_s(t) \end{aligned} \tag{10}$$

The following is the suggested SISO Add-On controller, which in our approach is a simple model reference adaptive controller (SAC).

$$u(t) = [k_1 + K_e(t)] e_a(y) + K_x(t) x_m(t) + K_u(t) u_m(t) \tag{11}$$

The adaptive gains are obtained as a combination of "proportional" and "integral" gains

$$K_e(t) = K_{Pe}(t) + K_{Ie}(t)$$

$$K_x(t) = K_{Px}(t) + K_{Ix}(t) \quad (12)$$
$$K_u(t) = K_{Pu}(t) + K_{Iu}(t)$$

$$K_{Pe}(t) = e_a e_a^T \Gamma_{Pe}$$
$$K_{Px}(t) = e_a x_m^T \Gamma_{Px} \quad (13)$$
$$K_{Pu}(t) = e_a u_m^T \Gamma_{Pu}$$

$$\dot{K}_{Ie}(t) = e_a e_a^T \Gamma_{Ie} - \sigma K_{Ie}(t), \quad K_{Ie}(0) = 0,$$
$$\dot{K}_{Ix}(t) = e_a x_m^T \Gamma_{Ix} \quad (14)$$
$$\dot{K}_{Iu}(t) = e_a u_m^T \Gamma_{Iu}$$

and where all Γ's' and σ are positive definite matrices of proper dimension or positive scalar, respectively. Figure 4 presents a detailed block diagram of the proposed Add-On adaptive control algorithm.

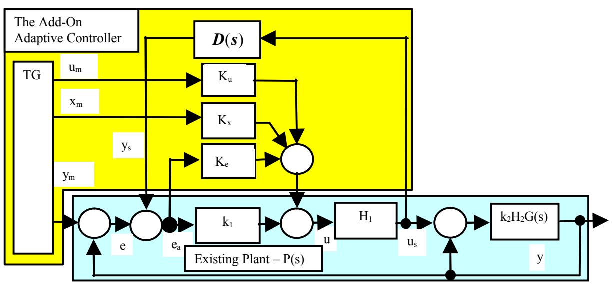

Figure 4: A detailed Block diagram of the proposed Add-On Adaptive controller for stable plant cascaded with PID controller.

5. Background and the Theory of the Add-on Controller – The SAC Algorithm

This section presents the background behind the rational of the Add-On SAC controller.

5.1 Definitions

We deal with transfer functions that are not necessarily proper rational functions. For the sake and convenience of the mathematical generality, we define as follows:

1. A rational (not necessarily proper) function is stable if all its poles are in the **open** LHP.
2. A rational function is minimum phase if all its zeros are in the **open** LHP (otherwise it is non-minimum phase).
3. Relative degree, r: r = degree of the denominator – degree of the numerator.
4. $\tilde{S}$ Family of all stable, not necessarily proper, real-rational functions.
5. S Family of all stable, proper, real-rational functions.
6. $\tilde{S}_r, \tilde{S}_r$ above families, restricted to relative degree r.

For stating the stability and performance of the Add-On Adaptive Control two concepts are required:

a. Strictly Positive Real system (SPR);
b. Almost Strictly Positive system (ASPR).

Because the meaning of these concepts in real world systems is not very clear, a co-paper presented at this conference presents a clear picture of their definition and understanding.

In this work we assume that the plant $G_p(s)$

(i) Is a stable system;
(ii) is strictly proper, i.e. the relative degree = r ≥ 1.

5.2 Stability and performance

Rigorous proofs of stability and performance of the Add-On adaptive control algorithm presented in section 4 are presented in [2,3]. Here we only state the sufficient condition. The Add-On adaptive control algorithm is stable and the tracking error e_a asymptotically converges if the Parallel Feedforward Configuration (PFC), $D(s)$, is such that the augmented plant

$$P_a(s) = H_1(s)P(s) = H_1(s)\big(G_p(s) + D(s)\big) \quad (15)$$

is Almost Strictly Positive system(ASPR). (As we show below, it implies that if $D^{-1}(s)$ stabilizes $G_p(s)$, then $P_a(s)$ above is minimum phase. Then, if the relative degree of $P_a(s)$ is 1, the required ASPR conditions are satisfied.)

6. Parameterization of PFC that Renders Plants with PID controller ASPR

Here we deal with the following issue: When can a plant be converted into an ASPR plant? We only deal here with stable systems cascaded with PID controller. The main result that is formalized in this section is that any stable system cascaded with PID controller can be augmented in such a way as to make it ASPR.

We will parameterize a set of parallel feed-forward configurations (PFC) that convert any stable system cascaded with PID controller into an ASPR plant.

Here, we use ideas from [2,5]:

Theorem 1: Consider the system $G_p \in S$ (stable and proper). The set of all controllers, C, for which the feedback system is internally stable equals

$$\left\{ \frac{Q}{1-QG_p}; Q \in S \right\} \quad (16)$$

Proof: The following is a short sketch (see[5] for details):

$$\frac{CG_p}{1+CG_p} = \frac{\frac{Q}{1-QG_p} G_p}{1 + \frac{Q}{1-QG_p} G_p} = \frac{QG_p}{1 - QG_p + QG_p} = QG_p \in S$$

Q.E.D.

Notice:

i) **C** is not necessarily stable or minimum phase!

ii) in [5] one can find more general parameterizations of the set of all stabilizing controller for any proper plant.

Theorem 2: [2]

ASPR$\Leftrightarrow$minimum phase + relative degree$\in \{0,1\}$.

Theorem 3: Consider the system $G_p(s) \in S$ (stable+ proper). Let $Q \in \tilde{S}_{-1} \bigcup \tilde{S}_0$ (i.e. stable+relative_degree$\in \{0,-1\}$).
If $D^{-1} = \frac{Q}{1-QG_p}$ then $G_p(s)+D(s)$ is minimum_phase and relative_degree$\in \{0,1\}$ and thus ASPR.
Proof: [6] We have (a short sketch)

$$G_p + D = G_p + \frac{1-QG_p}{Q} = \frac{QG_p + 1-QG_p}{Q} = \frac{1}{Q}. \quad \text{Q.E.D.}$$

Theorem 3a: Assume that the system $G_p \in S$ (stable+ proper). Let Q^{-1} be minimum phase + relative_ degree $\in \{0,1\}$). If $D(s) = Q^{-1}(s) - G_p(s)$ then $G_p(s)+D(s)$ is minimum phase + relative degree $\in \{0,1\}$ and thus ASPR.
Proof: see [6].

Lemma 1: The PID controller $H_1(s)$ in (4) is minimum phase and relative degree=0, therefore is ASPR.
Proof: [7].

As stated in section 5.2 we need (14) to be ASPR. This is formulated in the following.
Theorem 4: The augmented plant
$P_a(s) = H_1(s)\left(G_p(s)+D(s)\right)$ is ASPR.
Proof: [7].

Remarks:
(i) Theorems 4 states that for any stable system cascaded with PID controller there always exists ASPRing PFC, denoted $D(s)$.
(ii) The converse of Theorem 4 is not correct. That is, Theorems 4 does not parameterize all PFC's that are ASPRing stable system cascaded with PID controller.
(iii) If in addition Q the theorems 3 and 3a is minimum phase then $G_p(s)+D(s)$ is stable as well.
(iv) We restricted Theorems 3 and 3a to stable systems and Theorem 4 for stable plant cascaded with PID controller only because of its simplicity and its direct applicability to many real world problems. However, in general, ASPR plants that are created by parallel feedforward configuration (PFC) do not require the original plant to be stable are not required to be minimum phase.

7. Parallel Feedforward Configuration (PFC) in Practice

In this section we show a practical how-to implementation of Parallel-Feedforward Configuration (PFC). From the derivation in the preceding section, one may question the applicability of PFC. One may even ask: How do you add something to the plant output? Do you bend the motor axis?
The following shows that the concept of PFC is implementable as in Figure 4, i.e. in the Add-On adaptive control algorithm itself.
First, Figure 5 shows the Add-On control algorithm for the case that the plant in cascade with the PID controller $H_1(s)P(s)$ is ASPR.

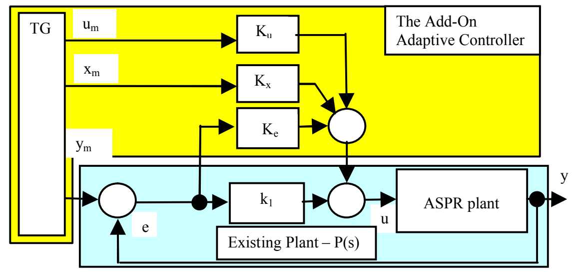

Figure 5: The Add-On SAC algorithm for an ASPR plant.

When $G_p(s)$ is stable but $H_1(s)G_p(s)$ is not ASPR, we showed in theorem 4 that when $D(s)$ is selected according to Theorem 3 or 3a, then, the augmented plant $P_a(s)$ is ASPR.
Figure 6 shows the Add-On adaptive control algorithm for this case.

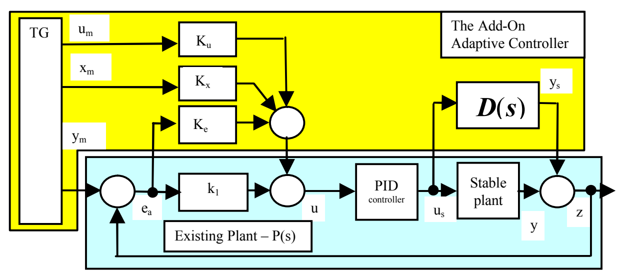

Figure 6: The SAC algorithm for stable plant cascaded with PID controller.

We now apply the following operations

$$\begin{aligned} u &= K_e(y_m - y_s) + K_x x_m + K_u u_m + k_1(y_m - y) \\ &= K_e\left[y_m - (y + Du_s)\right] + K_x x_m + K_u u_m + k_1(y_m - y) \quad (17) \\ &= K_e(y_m - y - Du_s) + K_x x_m + K_u u_m + k_1(y_m - y) \end{aligned}$$

The first row is direct implementation of PFC on the ASPRed plant. The last row is a different mechanization of the Add-On SAC algorithm where the PFC is implemented within the algorithm as depicted in Figure 4.
We note that PFC allows taking plants of any relative degree and even non-minimum phase, and make them into ASPR plants. However, as instead of $P(s)$ one now controls $P_a(s)$, even when one can obtain ideal performance for $H_1\left(G_p + D\right)$, the added $D(s)$ is necessary ballast, needed to guarantee stability with adaptive controllers. We note that if $D(s)$ is small, its effect on the

performance is also small.

8. Example

A great amount of examples are presented in the literature. In this example we assume

$$k_2 H_2 G(s) = \frac{s+4}{s^3+5s^2+6s} \quad and \quad k_1 = 500.$$
$$H_1(s) = 1 + \frac{0.1}{s} + \frac{0.5s}{0.1s+1} \tag{18}$$

8.1 Existing Performance

The existing performance is to track a trajectory that is shaped by a TG with time constant of 1 sec that is fed by a square wave with frequency of 0.2 Hz. The performance, that has been satisfactory for the present system, is presented in Figure 7.

8.2 Present Performance with Updated Requirement

The new requirement is to track a trajectory that is shaped by a TG with time constant of 0.1sec that is fed by a square wave with frequency of 0.4 Hz. The application of this new input to the existing system gives unacceptable performance, as presented in figure 8.

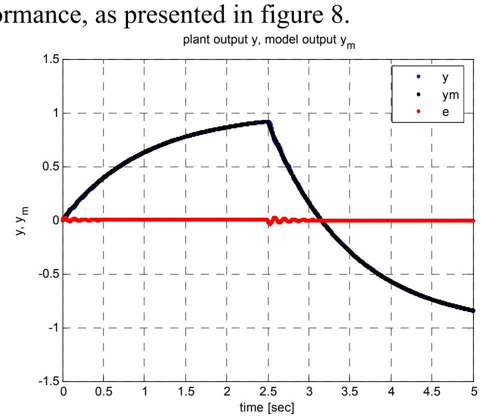

Figure 7: Model output, plant output, and tracking error – M(s)=1/(s+1).

8.3 Performance with Add-On Adaptive Controller

To apply the proposed Add-On Adaptive Controller we select a minimum phase $Q(s)$ with relative-degree=-1

$$Q(s) = \frac{100(s+0.5)(s^3+5s^2+7s+4)}{s^3+105s^2+457.5s+204} \tag{19}$$

Then from theorem 3a

$$D(s) = Q^{-1}(s) - P(s) = \frac{1}{100(s+0.5)} \tag{20}$$

We need to implement only $D(s)$ in (8). Further analysis presented in the co-paper [7] shows that the ASPR conditions are satisfied. The system is required to track the output of a purposely selected reduced order model (TG) of order 1, M(s)= 1/(0.1s+1), as shown in Figure 9 (in black). The new requirement is to track a square wave of 0.4Hz. Figure 9 shows as well the output of the plant (in blue) and the tracking error with the proposed Add-On Adaptive Controller.

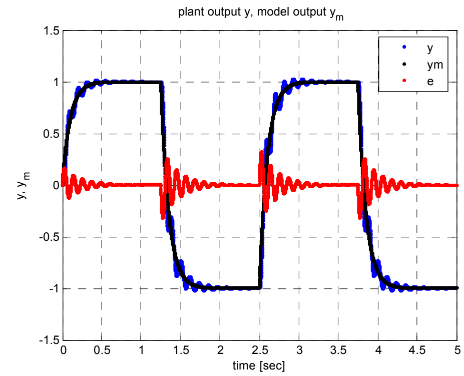

Figure 8: Model output, plant output and tracking error for high performance requirement - M(s)=1/(0.1s+1).

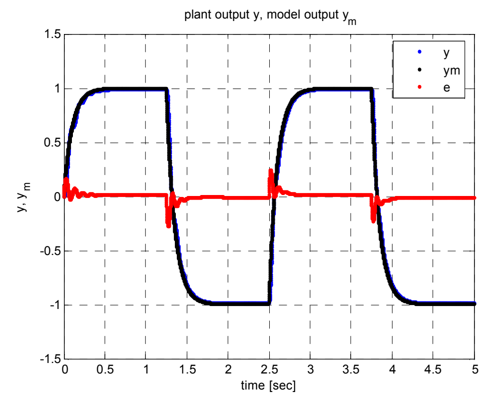

Figure 9: Model output, plant output and the tracking error with the Add-On adaptive controller.

Figure 10 shows the Add-On adaptive controller gains. We must also note, however, that the adaptive gain $K_{Ie}(t)$ in (14) would increase whenever the tracking error is not zero. Although it is proved [2,3] that all adaptive gains converge to constant finite values under ideal conditions, the gain $K_{Ie}(t)$ without the sigma-term in (14) would continually increase in the presence of any noise, even at those noise levels that are negligible for any other

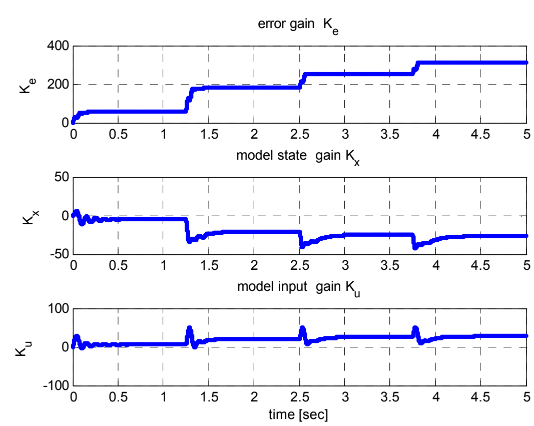

Figure 10: The gains of the Add-On Adaptive Control Algorithm.

practical purposes. Although an ASPR system remains stable with arbitrarily high gains, these gains may be too high for any practical (and numerical) purpose and may even diverge in time. This is the reason for the adoption of Ioannou's simple idea [8,9] and addition a sigma-term in(14) (or forgetting factor).This effect is not felt by the control gains $K_{Ix}(t)$ and $K_{Iu}(t)$ that move up-and-down according to the specific situation. With the sigma-term the error gain increases whenever it is required to increase (because of large errors, etc.) and decreases when large gains are not needed any more. The coefficient σ can be very small, because its aim is only to prevent the gain from increasing without bound. Figure 11 presents a block diagram of the example's existing plant and Add-On adaptive control algorithm that has been used to derive the results in this paper. This demonstrates the simplicity of the presented Add-On Adaptive Control Algorithm.

9. Conclusions

An Add-On adaptive control algorithm that can always improve the performance of any system controlled by PID controller has been presented. The proofs are constructive, thus presenting simple cost-effective alternative to requirement of improving system performance.

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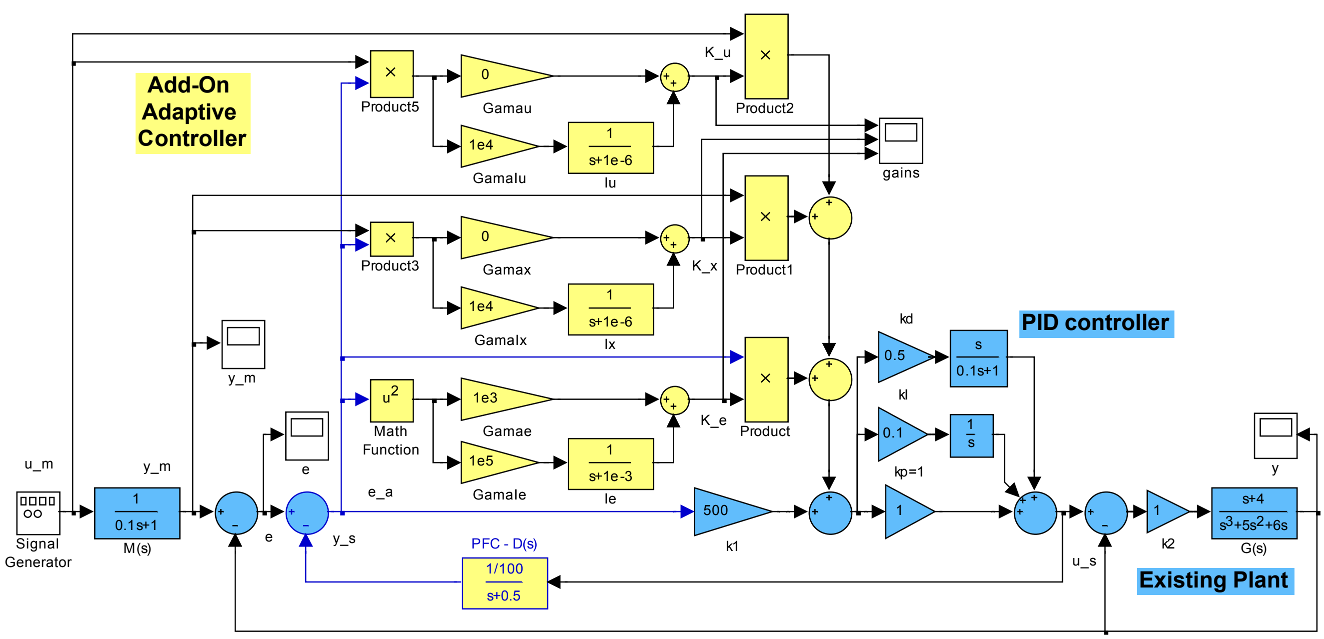

Figure 11: Block diagram of Simulink program of the example's existing plant and Add-On adaptive control algorithm.